Chapter 3 Equilibrium

Lect. # 14,15 &16

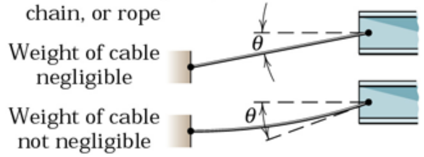
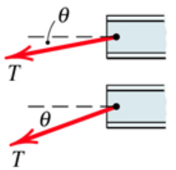
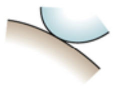
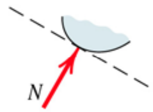
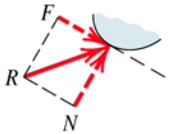
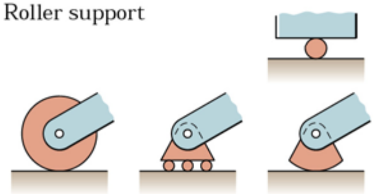
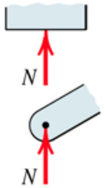
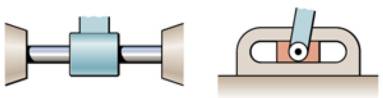
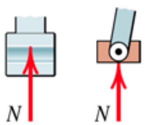
$R = \Sigma F = 0 \qquad M = \Sigma M = 0$

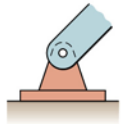
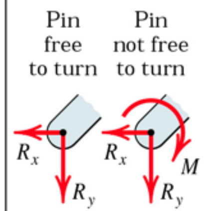
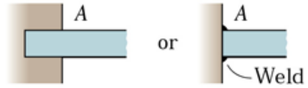
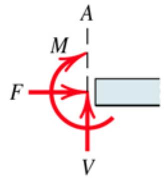
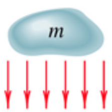
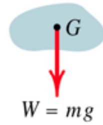
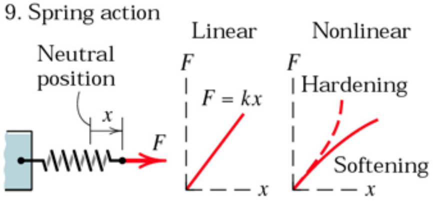
Necessary and sufficient conditions for equilibrium

Free-Body Diagram (FBD)

FBD is a diagrammatic representation of the isolated body (or combination of bodies treated as a single body) showing all forces applied to it by mechanical contact with other bodies that are imagined to be removed. If significant body forces are present then these forces must then be shown.

Modeling the action of forces

MODELING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS	
Type of Contact and Force Origin	**Action on Body to Be Isolated**
1. Flexible cable, belt, chain, or rope 	 Force exerted by a flexible cable is always a tension away from the body in the direction of the cable.
2. Smooth surfaces 	 Contact force is compressive and is normal to the surface.
3. Rough surfaces 	 Rough surfaces are capable of supporting a tangential component F (frictional force) as well as a normal component N of the resultant contact force R.
4. Roller support 	 Roller, rocker, or ball support transmits a compressive force normal to the supporting surface.
5. Freely sliding guide 	 Collar or slider free to move along smooth guides; can support force normal to guide only.

MODELING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS (*cont.*)	
Type of Contact and Force Origin	**Action on Body to Be Isolated**
6. Pin connection 	 A freely hinged pin connection is capable of supporting a force in any direction in the plane normal to the axis; usually shown as two components R_x and R_y. A pin not free to turn may also support a couple M.
7. Built-in or fixed support 	 A built-in or fixed support is capable of supporting an axial force F, a transverse force V (shear force), and a couple M (bending moment) to prevent rotation.
8. Gravitational attraction 	 The resultant of gravitational attraction on all elements of a body of mass m is the weight $W = mg$ and acts toward the center of the earth through the center mass G.
9. Spring action 	 Spring force is tensile if spring is stretched and compressive if compressed. For a linearly elastic spring the stiffness k is the force required to deform the spring a unit distance.

The force exerted by a contacting body or supporting member is always in the sense to oppose the movement of the isolated boby which would occur if the contacting or supporting body were removed.

Construction of Free-Body Diagrams

1) Decide on the body to be isolated (or cut).
2) Isolate the body by drawing a diagram which represents its complete external boundary.
3) Identify all forces which act on the isolated body.

Generally, these are due to:

Applied loading

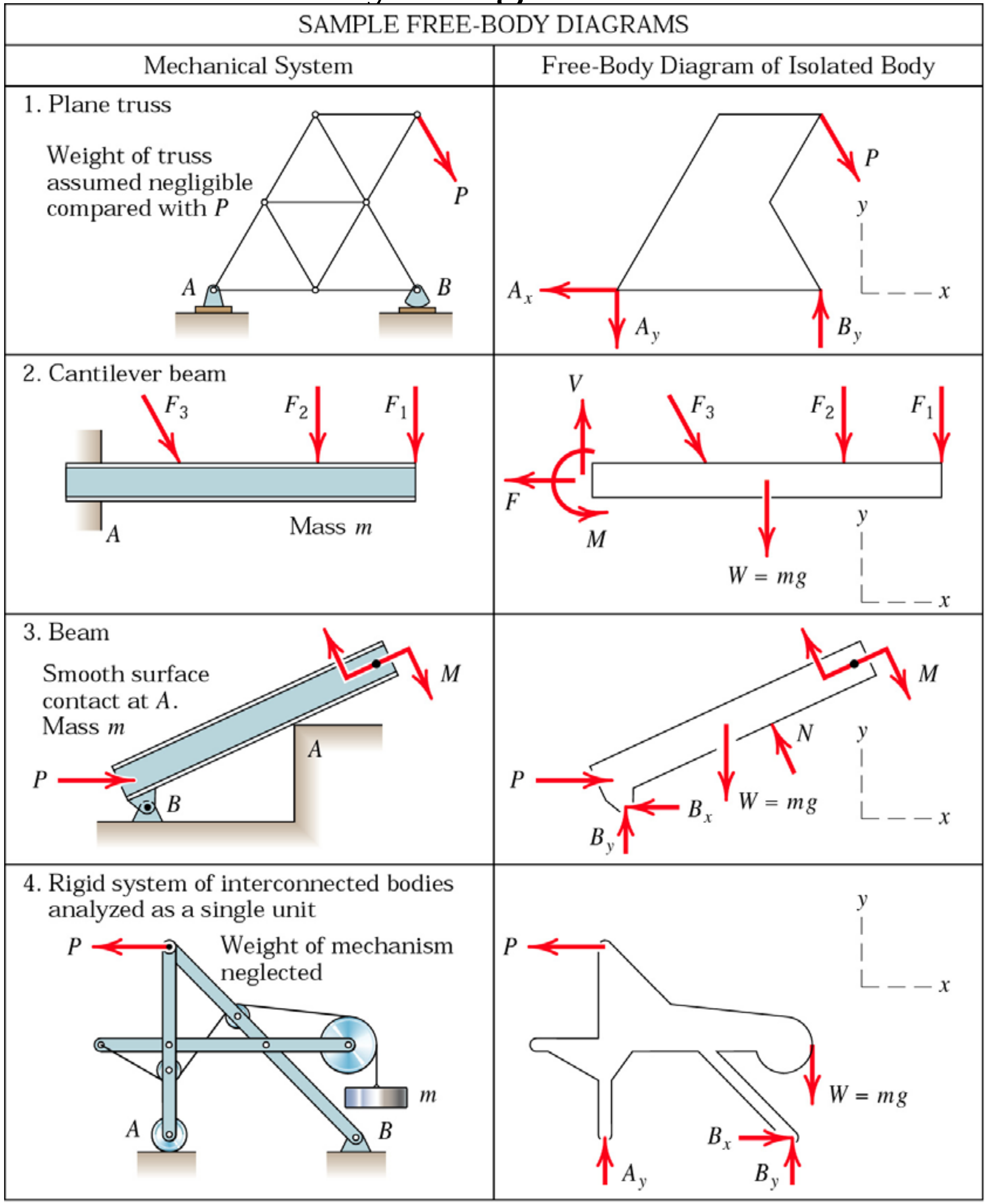
Reactions

Weights if appreciable

Unknown forces should be represented by a vector arrow with the unknown magnitude or direction indicated by symbol. The subsequent calculations with the equilibrium equations will yield a positive quantity if the correct sense was assumed and negative quantity if the incorrect sense was assumed.

4) Show the choice of the coordinate axes directly on the diagram.

Examples of Free-body Diagrams

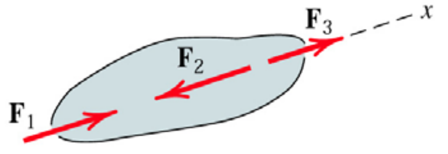
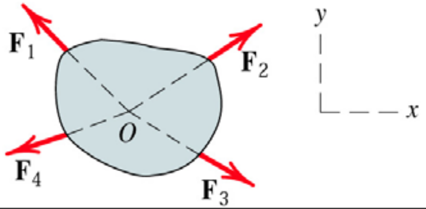
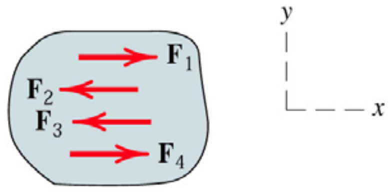
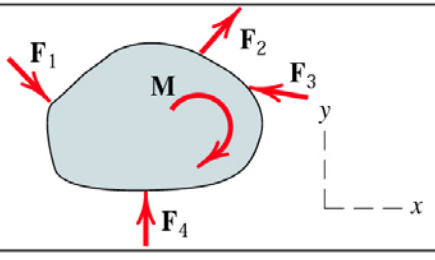


Equilibrium Conditions

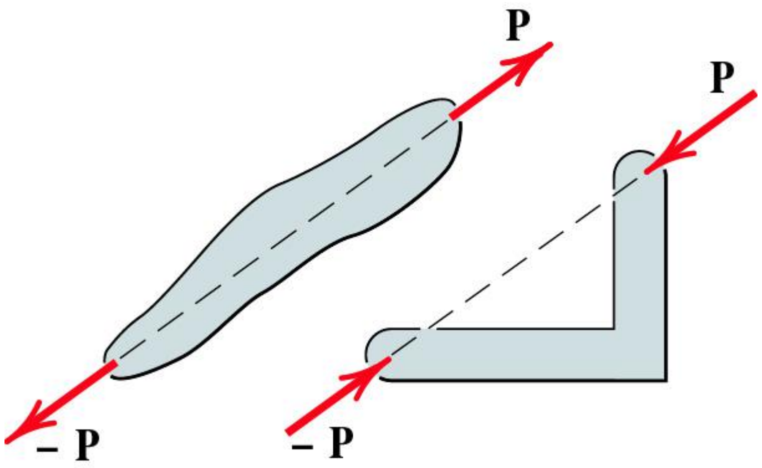
$$\mathbf{R} = \Sigma \mathbf{F} = 0 \qquad \mathbf{M} = \Sigma\, \mathbf{M} = 0$$

2D

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma M_o = 0$$

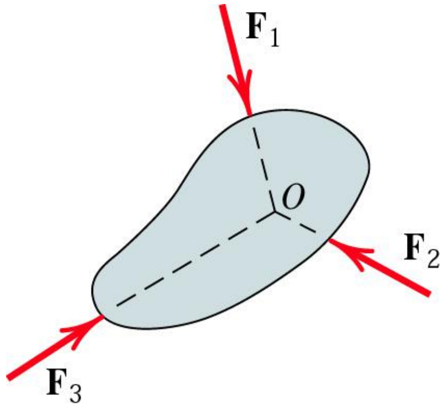
CATEGORIES OF EQUILIBRIUM IN TWO DIMENSIONS		
Force System	Free-Body Diagram	Independent Equations
1. Collinear		$\Sigma F_x = 0$
2. Concurrent at a point		$\Sigma F_x = 0$ $\Sigma F_y = 0$
3. Parallel		$\Sigma F_x = 0 \quad \Sigma M_z = 0$
4. General		$\Sigma F_x = 0 \quad \Sigma M_z = 0$ $\Sigma F_y = 0$

- Two- and Three-Force Members

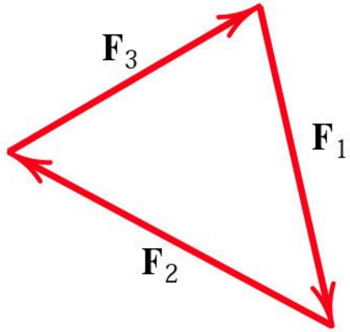


Two-force members

Equal, opposite
and collinear



(*a*) Three-force member



(*b*) Closed polygon satisfies $\Sigma\mathbf{F} = \mathbf{0}$

Constraints and Statical Determinacy

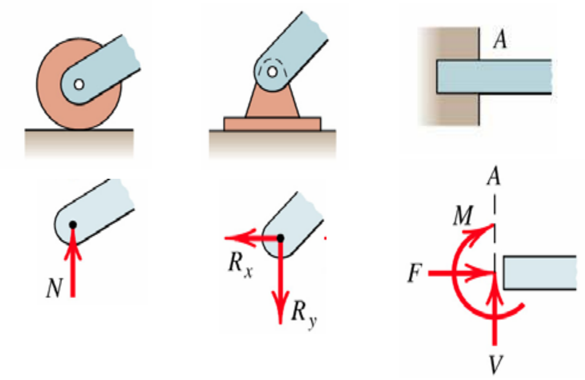
Equilibrium Equations

$\Sigma F_x = 0 \quad \Sigma F_y = 0 \quad \Sigma M_o = 0$

3 equations

External Supports

unknowns



By constraint we mean the restriction of movement

Statically Determinate

 # Equilibrium Equations = # of Unknowns

Statically Indeterminate

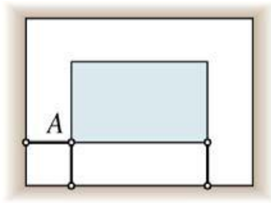
 # Equilibrium Equations < # of Unknowns

- Adequacy of Constraints

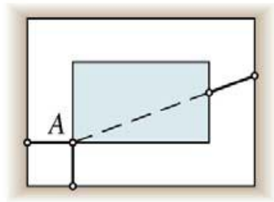
Adequate

Partial

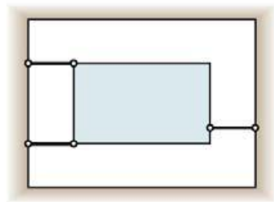
Redundant



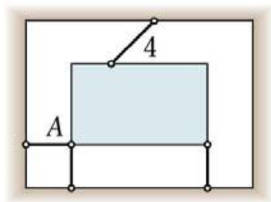
(*a*) Complete fixity
Adequate constraints



(*b*) Incomplete fixity
Partial constraints



(*c*) Incomplete fixity
Partial constraints



(*d*) Excessive fixity
Redundant constraint

Equilibrium in Three Dimensions *(optional)*

$$\Sigma \mathbf{F} = \mathbf{0}$$

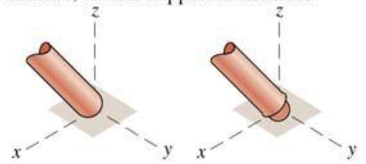
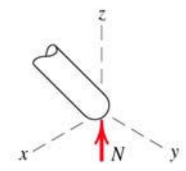
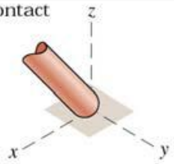
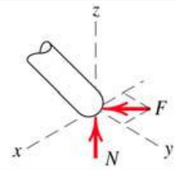
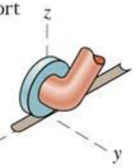
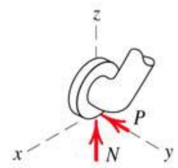
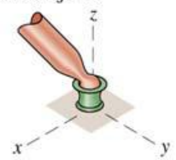
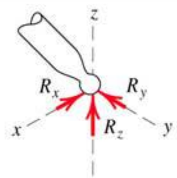
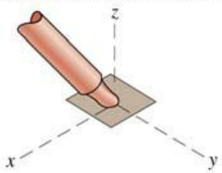
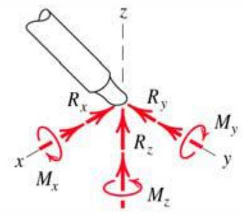
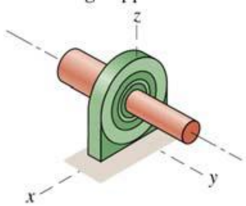
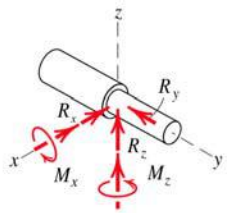
$$\Sigma\ \mathbf{M} = \mathbf{0}$$

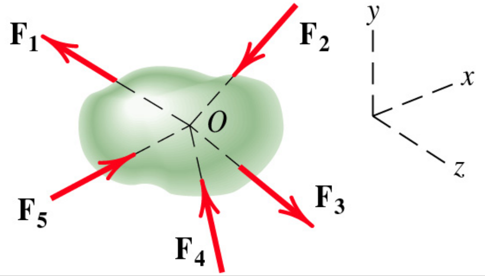
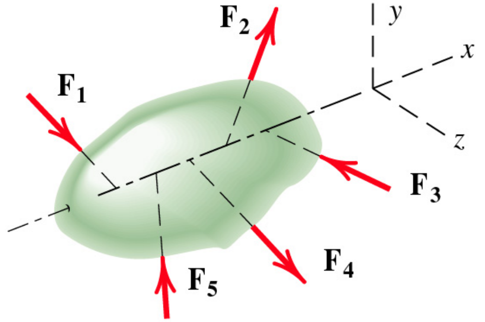
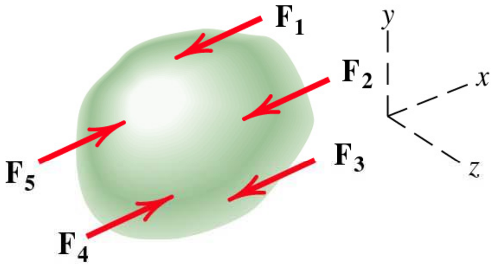
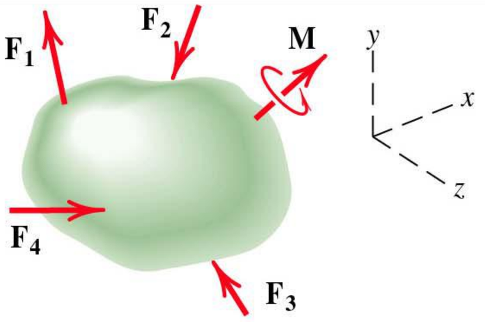
or

$$\Sigma F_x = 0 \quad \Sigma F_y = 0 \quad \Sigma F_z = 0$$

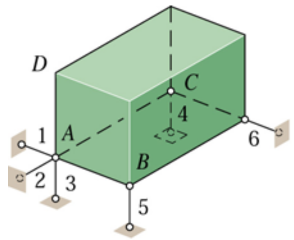
$$\Sigma M_x = 0 \quad \Sigma M_y = 0 \quad \Sigma M_z = 0$$

Free-Body Diagrams

MODELING THE ACTION OF FORCES IN THREE-DIMENSIONAL ANALYSIS		
Type of Contact and Force Origin	Action on Body to Be Isolated	
1. Member in contact with smooth surface, or ball-supported member 		Force must be normal to the surface and directed toward the member.
2. Member in contact with rough surface 		The possibility exists for a force F tangent to the surface (friction force) to act on the member, as well as a normal force N.
3. Roller or wheel support with lateral constraint 		A lateral force P exerted by the guide on the wheel can exist, in addition to the normal force N.
4. Ball-and-socket joint 		A ball-and-socket joint free to pivot about the center of the ball can support a force **R** with all three components.
5. Fixed connection (embedded or welded) 		In addition to three components of force, a fixed connection can support a couple **M** represented by its three components.
6. Thrust-bearing support 		Thrust bearing is capable of supporting axial force R_y as well as radial forces R_x and R_z. Couples M_x and M_z must, in some cases, be assumed zero in order to provide statical determinacy.

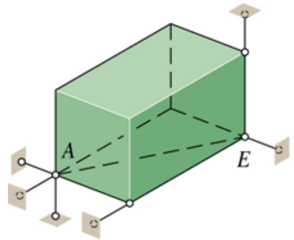
CATEGORIES OF EQUILIBRIUM IN THREE DIMENSIONS		
Force System	Free-Body Diagram	Independent Equations
1. Concurrent at a point		$\Sigma F_x = 0$ $\Sigma F_y = 0$ $\Sigma F_z = 0$
2. Concurrent with a line		$\Sigma F_x = 0$ $\Sigma M_y = 0$ $\Sigma F_y = 0$ $\Sigma M_z = 0$ $\Sigma F_z = 0$
3. Parallel		$\Sigma F_x = 0$ $\Sigma M_y = 0$ $\Sigma M_z = 0$
4. General		$\Sigma F_x = 0$ $\Sigma M_x = 0$ $\Sigma F_y = 0$ $\Sigma M_y = 0$ $\Sigma F_z = 0$ $\Sigma M_z = 0$

Constraints and Statical Determinacy



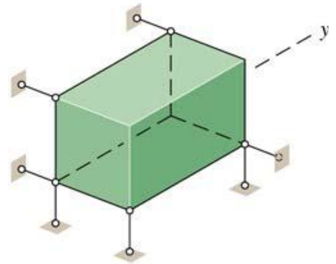
(a) Complete fixity
Adequate constraints

no resistance to moment about line AE

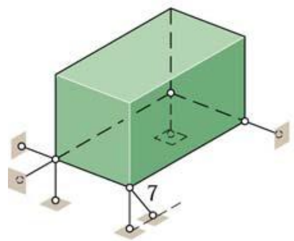


(b) Incomplete fixity
Partial constraints

No resistance to the unbalanced force in the y direction



(c) Incomplete fixity
Partial constraints



(d) Excessive fixity
Redundant constraints