Improving Costas Radar Signal by Applying Variable Time Spacing Using Costas Array And Golomb Ruler [1]

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Abstract

The normal Costas frequencies code sequence signal is very well known and important signal in pulse compression Radar. In this paper we present two algorithms to modify Costas sequence by arranging the frequencies of Costas signal in time, first using binary Costas array and second using Golomb Ruler. These methods enable us to control side-lobes and to improve Doppler frequency resolution of Ambiguity Function (AF).

At first we present the principle of these methods Golomb Ruler and Costas array. Then we apply these two methods to normal Costas signal, modified Costas signal and step frequency modulation and calculate the AF for all.

The results of comparison have shown that considerable reduction of side-lobes of AF is achieved by using these two methods, and consequently an improvement of AF is obtained.

Keywords: AF: Ambiguity Function, Costas signal, ACF: Autocorrelation Function, Golomb rulers, LFM: Linear Frequency Modulation.

[1]For the abstract in Arabic see page (31) .

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1- Introduction

The essential part of improving radar system performance is to develop new radar signal. Consideration is given to the effect of delay resolution, Doppler resolution, noise immunity, and intentional and unintentional interference suppression. The aforesaid requirements have caused to develop these signals assisted by modern signal processing systems.

In general for good detection most Radar seeks to transmit long-duration pulses to achieve high energy, since transmitters are typically operated near their peak power limitation. On the other hand, for good range measurement accuracy, radar needs short pulses. These divergent of the needs of long pulses for detection and short pulses for range resolution in measurements prevented early radars from simultaneously performing both functions. Fortunately, in the late 1950s and early 1960s a new concept was developed whereby both needs could be met. The concept is called ***pulse compression***. It makes use of the fact that a long-duration pulse's bandwidth can be made larger by use of Frequency Modulation (FM). Large bandwidth implies narrow effective duration. With FM a waveform can be designed to have both long pulse duration and short pulse effective duration (large bandwidth). The waveform with short pulse effective duration is produced when the long-duration waveform with FM is applied to its matched filter for receiving pulses. Thus, by using FM over long transmitted pulses and a matched filter, a system can simultaneously obtain good performance for detection and range resolution and accurate range measurements.

The researchers develop many ***pulse compression*** Radar signals assisted by modern signal processing systems. Consequently, signals in different shapes have been presented like **Phase coded signals** such as Barker Code, Frank Code, P1, P2, P3, Px Codes, as well as m-sequence code, Colombo code, etc., and **frequency coding** such as Costas signal [4,6,7]. Each of these signals has advantages and disadvantages. The most important one in the frequency coding is Costas signal.

Costas sequences are generally used in the design of frequency-coded waveform, which ensure high delay-Doppler Resolution. An important property of Costas sequence is that a sequence of length N when used in the radar signal design would yield an **A**mbiguity **F**unction (AF) with side-lobes of maximum height 1/N times of its main-lobe height.

In radar scenario, no waveform is optimum for target resolution in general. On the other hand, an optimum ambiguity surface should be of a sharp central spike surrounded by a clear area with no volume, then when the bulk of the volume pushed away from the central peak, then the interference can be avoided [1,2].

In this direction, by adding variable time spacing between sub-pulses of Costas sequence we will improve the AF of Costas sequence by reducing side-lobes and push away the maximum side-lobes peaks from the central peak without increasing the number of frequency and without increasing frequency spacing in Costas sequence. In this paper we suggested two algorithms to arrange the frequencies in time to enable us to control the side-lobes by using Golomb ruler or Costas array.

Costas pulse T consists of N sub-pulses; each sub-pulse has different frequency modulation as shown in Fig. 1-a. [2], [4]. Each frequency is chosen from a series of frequencies within the bandwidth B. We have N frequencies, each frequency is multiple of $\Delta f = 1/t_b$, and pulse width of each sub-pulse is given by $t_b = T/N$. Costas has suggested algorithm to arrange the frequencies to enable us to control the side-lobes in such a way that these side-lobes will not exceed 1/N. Then, the biggest side-lobes in AF is 1/N of its value at the main-lobe Fig.1-b and Fig.2. Costas signal has a delay resolution of $1/N^2$ and the Doppler resolution of 1/T and because of using a matched filter, the received signal has noise immunity. However, Costas pulse is not an optimum signal.

Reference [8] introduced "*a modified Costas signal*", which involves an increase of frequency separation Δf beyond the inverse of sub-pulse duration t_b. Upon this, they decrease the side-lobes of AF at Zero Doppler Cut within the range $\tau > t_b$, and the delay resolution is improved (The delay Resolution at Costas Signal is $1/N^2$, and it is nearly $1/N^2 * \Delta f * t_b$ when there is frequency separation), but unfortunately it reveals grating lobes and side-lobes at $\tau < t_b$. The number of grating lobes is increased when the frequency spacing of Costas Signal is increased. The second suggestion in this reference is by adding **L**inear **F**requency **M**odulation (LFM) to the sub-pulse of the Costas signal to overcome the grating lobes, which appear when increasing Δf , of AF at $\tau < t_b$.

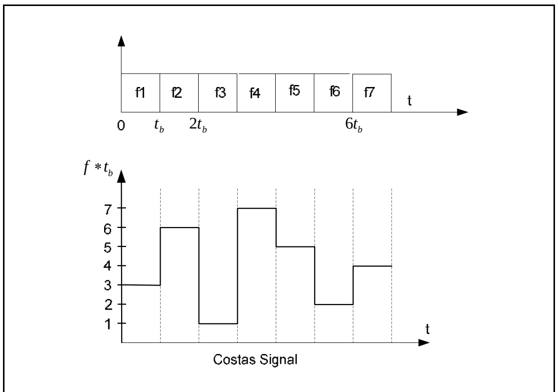
Reference [9] suggests another arrangement of the frequency-coded pushing sequence and also adding LFM to the sub-pulses which result in pushing the volume of AF outside the clear area.

Pulse Radar systems using variable time spacing between pulses, usually found in moving-target indicator (MTI). It is used for maximizing the radar's unambiguous range and Doppler while minimizing blind range and speed. They are called stagger and jitter pulse Radar. The reference [11] presents invention used time-hopping codes (THC) in ultra-wide band radar system (UWB) for interference rejection and suppression, and it proves that the variable time spacing can be able to modify the spectral properties of the pulse trains, and can achieve a desirable spectral response.

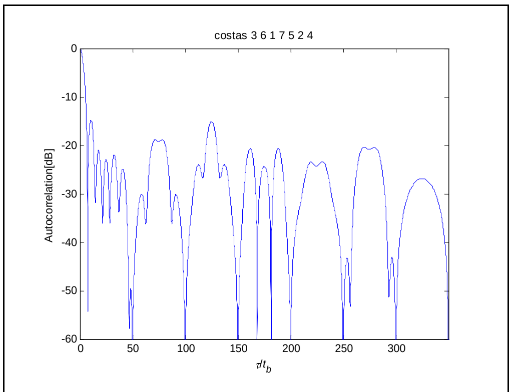
In this paper we will use variable time spacing between sub-pulses of Costas pulse signal to modify the Auto Correlation Function (ACF) (AF at zero Doppler cut) properties of the pulse. In general the ACF equals the inverse Fourier transform of the power spectral density thus we got the relationship:

Autocorrelation $\Leftrightarrow$ F^{-1} {power spectrum}

And we study the effect variable time spacing at ACF to enhance the performance specifications by reducing the side-lobes of the ACF at range $\tau > t_b$ without increasing Costas size or without increasing frequency spacing and without adding LFM to the sub-pulses.



a.



b.

Figure 1; a. Costas signal, b. ACF to normal Costas signal {3 6 1 7 5 2 4}

In Section 2, the basic concept of reduction the recurrent side-lobes of Costas sequence is shown. In Sections 3, 4 a brief description of techniques to reduce recurrent side-lobes of Costas sequence, Costas method, and Golomb method is given. In Section 5, we present applications of using Golomb rulers, Costas space time upon some of frequency coded signals, and a summary of results and comparisons between ACF of these signals before and after applying Golomb ruler, and Costas method. Finally, Section 6 presents a summery of results and conclusions.

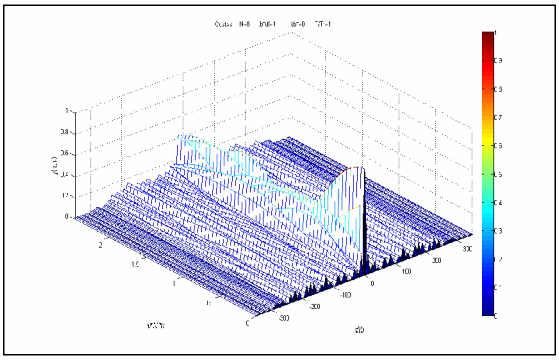


Figure 2 Ambiguity Function of Costas {3 6 1 7 5 2 4}

2- Principle of reducing recurrent side-lobes.

The principle of reducing the side-lobes of ACF of Costas sequence depends on studying ACF of the signal which is zero Doppler Cut of AF. The complex envelope of Costas signal (Fig. 3) whose hopping sequence $a=\{a_1, a_2, ..., a_N\}$ and fix time spacing between sub-pulses is given by:

$$u(t)=\frac{1}{\sqrt{N}}\sum_{n=1}^{N-1} u_n[t-(n-1)T_r], \quad (1$$

$$u_n(t)=\begin{cases}\exp(j2\pi f_n t), & 0\le t\le t_b \\ 0 & elsewhere\end{cases} \quad (2$$

$$f_n=\frac{a_n}{t_b}, \quad (3$$

- At $\tau < t_b$: The ACF of Costas signal with fix time spacing between sub-pulses is the same as ACF of the train stepped frequency [3] and [5]. It is independent of the order of pulses in time. This is achieved if the two signals have the same number of pulse train, the same frequency spacing, and the same spacing between pulses or sub-pulses (as long as the spacing between sub-pulses longer than the twice of pulse duration $T_r > 2t_b$).

The ACF of Costas signal with fix time spacing at $\tau \le t_b$ is the sum of the autocorrelation of each sub-pulse of reference signal and sub-pulse of the received signal which has the same frequency (Fig. 3-a) as fellows:

$$R(\tau)=\sum_{p=1}^{N} R_{u_p u_p}(\tau), \qquad |\tau|\le t_b \quad (4$$

where (5

$$R_{u_p u_p}(\tau)=\int_0^{t_b} u_p(t)u_p^*(t+\tau)dt,$$

This is yielding the main-lobe area of ACF.

- At $\tau > t_b$: The ACF of Costas sequence or stepped-frequency with fix spacing between sub-pulses (Fig. 3-b) consists of:

- Recurrent side-lobes: The recurrent side-lobes are at multiple of T_r.

$$R(\tau)=\sum_{p=n+1}^{N} R_{u_p u_{p-n}}(\tau), \quad \text{At range} \quad (6$$

$$nT_r - t_b < \tau < nT_r + t_b \qquad n=1,..,N-1$$

Then first recurrent side-lobes is at n=1 will be $R(\tau)=\sum_{p=2}^{N} R_{u_p u_{p-1}}(\tau)$,

The second recurrent side-lobes is at n=2 will be $R(\tau)=\sum_{p=3}^{N} R_{u_p u_{p-2}}(\tau)$,

and so on.

Because the complex envelopes $u_p(t)$ of different pulses have different center frequencies, the spectral overlap is relatively small, yielding ACF with recurrent lobes that are considerably lower than the main lobe.

The recurrent side lobes of Costas and stepped frequency signals are not the same because the recurrent side-lobes depend on frequencies which are crossed.

- Zeros between recurrent side-lobes occur if the spacing between sub-pulses is longer than the twice of pulse duration $T_r > 2t_b$.

In the next study, we will present two methods of reducing recurrent side-lobes of ACF by spreading them in all the range $\tau > t_b$ and not at specific places, where the cross correlation between the reference Costas signal and the received Costas signal at any time of $\tau > t_b$ have only one or two frequencies crossed together.

The way to achieve that is to spread the sub-pulses of Costas sequence by time by using variable spacing between sub-pulses. The complex envelope of Costas signal (Fig. 4) whose hopping sequence $a = \{a_1, a_2, ..., a_N\}$ and variable time spacing between sub-pulses is given by:

$$u(t) = \frac{1}{\sqrt{N}} \sum_{n=1}^{N-1} u_n[t - T_n], \quad (7$$

$$u_n(t) = \begin{cases} \exp(j2\pi f_n t), 0 \le t \le t_b, \\ 0 \quad elsewhere \end{cases} \quad (8$$

$$f_n = \frac{a_n}{t_b}, \quad (9$$

$$T_n = [T_1, T_2, ..., T_N], \;\; T_1 = 0, \quad (10$$

$$a_n = [a_1, a_2, ..., a_N], \quad (11$$

The ACF of Costas signal with variable spacing Fig. 4-a at $\tau \le t_b$ is the same as ACF of the equation (1), the shape of recurrent lobes of ACF at $\tau > t_b$ are different in time and level than the recurrent side-lobes, of the Costas sequence with fix spacing. First recurrent side-lobes of the ACF at $2t_b - t_b > \tau > 2t_b + t_b$ **is**

$$R(\tau) = R_{u3u2},$$

Second recurrent side-lobes is at $3t_b - t_b > \tau > 3t_b + t_b$ will be

$$R(\tau) = R_{u4u2},$$

Each recurrent side-lobes of ACF is a cross correlation between one sub-pulse of reference signal and one sub-pulse of the received signal. This yield, recurrent lobes of ACF at any part of range $\tau > t_b$ is lower than the recurrent lobe of ACF of the Costas sequence with constant time spacing and zeros side-lobes will be reduced or removed. The shape of recurrent side-lobes of the ACF depends on the time spacing design and the Costas signal design.

Fig. 3 and 4 demonstrate schematically the difference alignments of received and reference sub-pulses for fix spacing and variable spacing, we find that the coincidence between the reference signal and received signal at constant spacing and variable spacing at $\tau < t_b$ Fig. 3-a, 4-a are the same, but coincidence between the reference signal and received signal at $\tau > t_b$ Fig. 3-b, 4-b are not the same where at 4-b there is only one pulse of reference signal u3 coincides with one pulse of received signal u2. But at 3-b there are many pulses of reference signal that coincide with received signal. There is no simple theoretical expression about ACF of Costas signal with variable time spacing then in our study we calculate the ACF numerically.

In the next section two methods of spreading the sub-pulses of Costas sequence is shown first by using binary Costas array and then by using Golomb rulers.

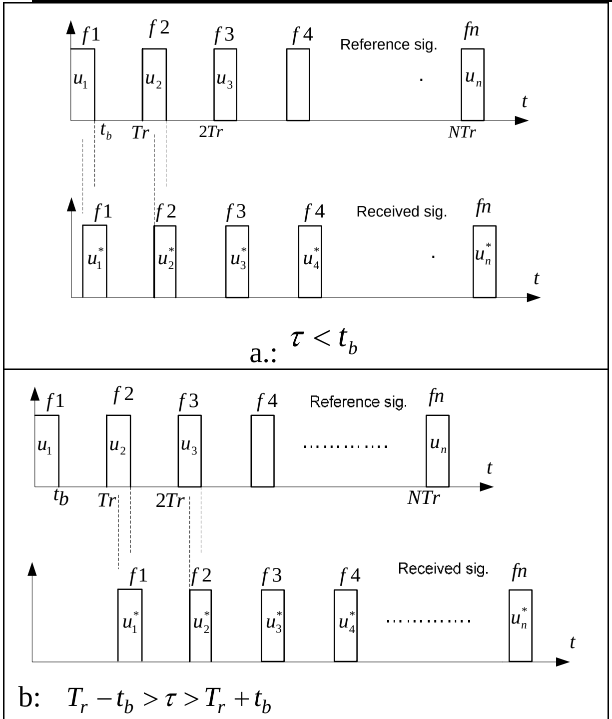


Figure 3. Costas with fix time spacing , a: at $\tau < t_b$; b: $\tau > t_b$

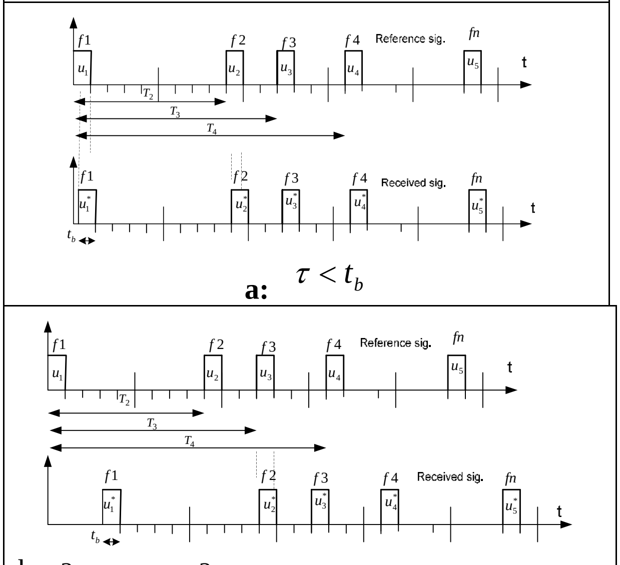


Figure 4. Costas with variable time spacing, a: at $\tau < t_b$; b: $\tau > t_b$

3- Method 1: Costas arrangement

This can be done by spreading the columns of binary Costas Array $N*N$ in time. Each cell in each column represent a unit time of t_b. At first we transmit the 1st column where one "1" represents a sub-pulse of width t_b and Zero "0" represents time spacing of length t_b, and second we transmit the 2nd column in the same way, and so on. Then the total time of sending Costas sequence is $N*N*t_b$ as shown in Fig. 5. This yield a train of Costas sub-pulses with variable time spacing between sub-pulses.

To demonstrate the effect of variable space time, we apply this method to many Costas sequences with different sizes and plot the AF, ACF. From plots of ACF shown in Fig. 6-a, and Fig. 6-b, we notice the following:

- From Fig. 6-a and Fig. 6-b we notice a reduction of the level of the recurrent side-lobes of ACF when compared with the recurrent side-lobes of ACF of Costas sequence with fix spacing and without spacing as shown in Fig.11-d and Fig. 11-a.
- Decrease the zero spacing in ACF when compared with ACF of Costas sequence with fix spacing at $\tau > t_b$.
- For $\tau < t_b$ The ACF of Costas signal with variable spacing is the same as ACF of Costas sequence with fix spacing.

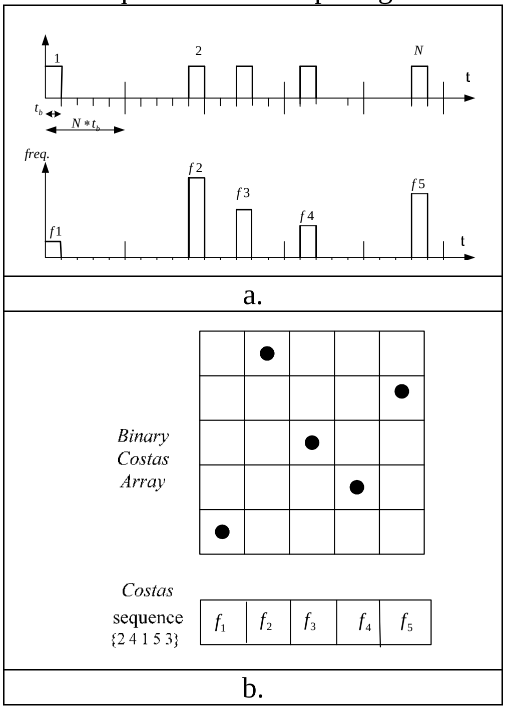


Figure 5: Costas with variable time spacing, a: Costas signal; b: Costas binary array.

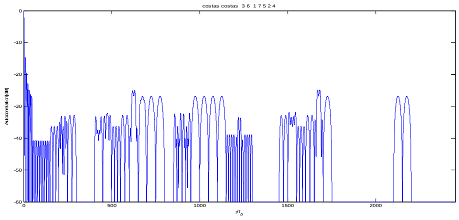


Figure 6-a. the ACF of Costas signal {3 6 1 7 5 2 4} with Costas array {3 6 1 7 5 2 4}.

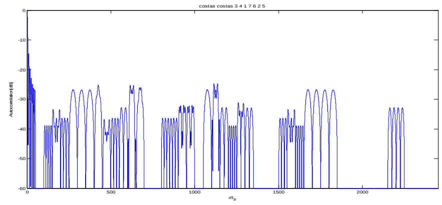


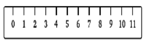
Figure 6-b. ACF of Costas signal {3 4 17 6 2 5} with Costas array {3 4 17 6 2 5}.

4- Method 2: Golomb ruler arrangement.

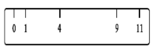
Golomb rulers are sets of positive integer numbers having all the differences between any pair of elements of the set to be unique.

Table. 1: Golomb rulers

n	length	position of marks
1	0	0
2	1	0 1
3	3	0 1 3
4	6	0 1 4 6
5	11	0 1 4 9 11 0 3 4 9 11
6	17	0 1 4 10 12 17 0 1 4 10 15 17 0 3 5 9 16 17 0 4 6 9 16 17
7	25	0 1 4 10 18 23 25 0 2 3 10 16 21 25 0 2 6 9 14 24 25 0 1 7 11 20 23 25 0 3 4 12 18 23 25
8	34	0 1 4 9 15 22 32 34



Common ruler



Golomb Ruler

Figure 7.

To demonstrate the effect of using Golomb rulers in Costas sequence, we apply Golomb ruler “0 3 4 12 18 23 25” to many of Costas sequences with the same size and plot the AF, ACF. From the plots AF and ACF of Fig 8, we notice the following:

- We notice a reduction of the level of the recurrent side-lobes of ACF when compared with the recurrent side-lobes of ACF of Costas sequence with fix spacing and without spacing as shown in Fig.9-c and Fig. 9-a .
- Most of zero spacing in ACF is removed when compared with ACF of Costas sequence with fix spacing at $\tau > t_b$.

These numbers can be thought of as ruler marks as an analogy with common ruler [10].

Using the Golomb ruler in Fig. 7 one can measure the distances {1; 2; 3; 4; 5; 7; 8; 9; 10; 11} by a suitable choice of two marks but no other distances can be measured. Moreover for each of these distances, only one pair of marks can be used to make such a measurement, therefore the Golomb property is satisfied. Some of known optimal Golomb rulers are shown in table 1.

We will use this property to form the modified Costas sequence. We set Golomb ruler at time axis and set the sub-pulses of Costas sequence at the marks of Golomb ruler, and then we obtain a train of Costas sub-pulses with variable time spacing between sub-pulses.

If for example the size of Costas sequence is N=7, we can use Golomb ruler with mark n=7 and length=25 and choose the sub-pulse of any position of marks n=7, example 0 3 4 12 18 23 25 or 0 2 6 9 14 24 25 etc.

- For $\tau < t_b$ The ACF Fig.8 is the same as ACF of Costas sequence with fix spacing Fig 9-c and Fig. 9-a .
- By good selection of frequency Costas sequence a low levels of side-lobes near the main-lobes of ACF can be obtained at range $\tau > t_b$.

Table 2 shows comparisons between the maximum values of side-lobes at $\tau > t_b$ of Costas sequence of N=7 without spacing (normal Costas), with fix spacing, with variable time using Costas spacing, and with variable time using Golomb ruler spacing.

Table 2: Maximum side-lobes of Costas signal without spacing, with fix spacing and with var. spacing.

Maximum Side-lobe

Costas	Normal Costas (dB)	Costas with fix spacing(dB)	Costas with Var. Spacing (Costas array) (dB)	Costas with Var. Spacing (Golomb Ruler) (dB)
2-1-6-4-7-3-5	-16.49	-18.97	-24.82	-26.86
2-1-5-7-3-6-4	-16.96	-19	-24.75	-26.86
3-6-1-7-5-2-4	-16.98	-18.92	-25	-26.86
2-4-7-3-1-6-5	-18.64	-21.99	-24.74	-26.86
3-4-1-7-6-2-5	-19.97	-21.99	-24.75	-26.86
7-1-3-6-4-5-2	-18.75	-20.99	-24.96	-26.86

From the table 2 we find that the side-lobes using Golomb rulers is the best of all sequences. But, the side-lobes when using fix space time gives zero level at range $t_b > \tau > 2t_b$ which is helpful in some applications.

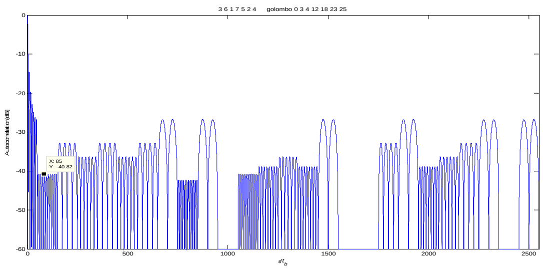


Figure 8-a. ACF of Costas signal {3 6 1 7 5 2 4} with Golomb spacing {0 3 4 12 18 23 25}.

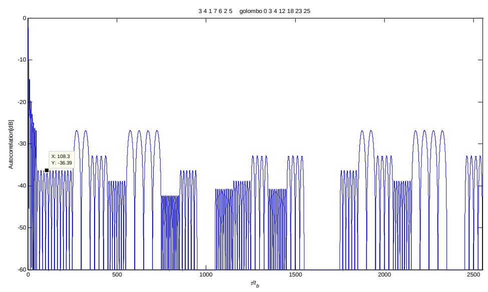


Figure 8-b. ACF of Costas signal {3 4 1 7 6 2 5} with Golomb spacing {0 3 4 12 18 23 25}.

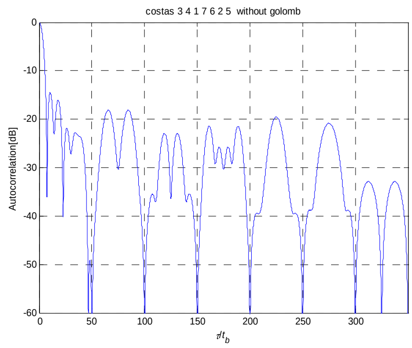
5- Application of using Golomb rulers and Costas space time in frequency coded signals.

a. Frequency coded: Costas sequence.

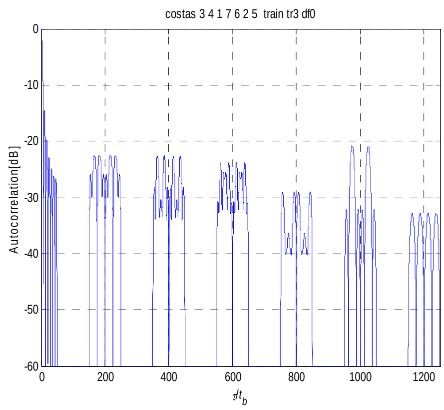
We applied Golomb ruler 0 3 4 12 18 23 25, and Costas time spacing to the frequency coded Costas sequence, with size N=7, frequencies {3 4 1 7 6 2 5} and $\Delta f = 1/t_b$. Each sequence consist of N sub-pulses, the total time for Costas sequence is $N * t_b$, the total time of Costas sequence with Golomb ruler spacing is $2T_G$, where T_G is the length of Golomb ruler, and the total time in Costas sequence with Costas time spacing is $N * N * t_b$.

The ACF of the above signals at $\tau < t_b$ are the same because the order of frequencies does not affect ACF. But the ACF at $\tau > t_b$ depends on the number of sub-pulses N, spacing between sub-pulses, and the order arrangements of frequencies.

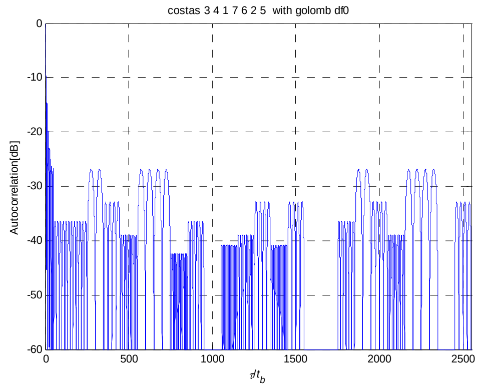
Comparing between the ACF of Costas sequence N=7, frequencies{ 3 4 1 7 6 2 5} and Golomb ruler 0 3 4 12 18 23 25 Fig. 9-b,with the ACF of Costas sequence N=7, frequencies { 3 4 1 7 6 2 5} Fig. 9-a, and with the ACF of Costas sequence N=7, frequencies { 3 4 1 7 6 2 5} and fix spacing $T_r = 3t_b$ Fig. 9-c, and with the ACF of Costas sequence N=7, frequencies {3 4 1 7 6 2 5} and Costas variable spacing Fig. 9-d we notice that the Costas sequence with Golomo ruler spacing time has the minimum side-lobes at $\tau > t_b$.



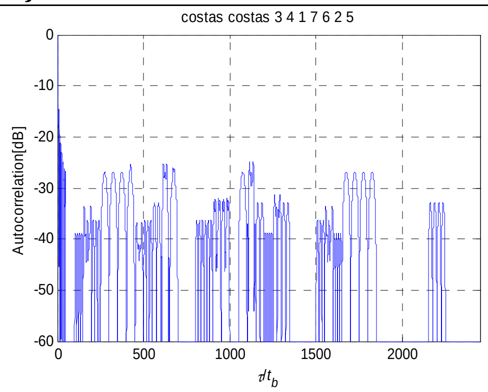
a. ACF of Costas sequence N=7, {3 4 1 7 6 2 5}



c. ACF of Costas sequence N=7, { 3 4 1 7 6 2 5} with fixd spacing $T_r = 3t_b$.



b. ACF of Costas sequence N=7,{3 4 1 7 6 2 5} with Golomb ruler 0 3 4 12 18 23 25

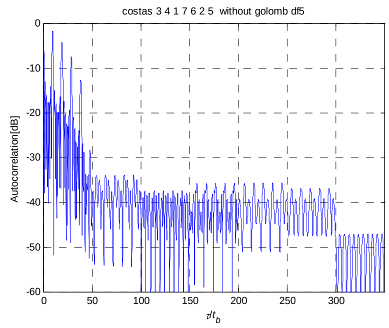


d. ACF of Costas sequence N=7, { 3 4 1 7 6 2 5}, with Costas variable spacing.

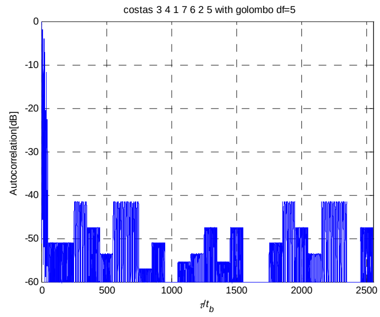
Figure 9

b. Frequency coded: Modified Costas sequence.

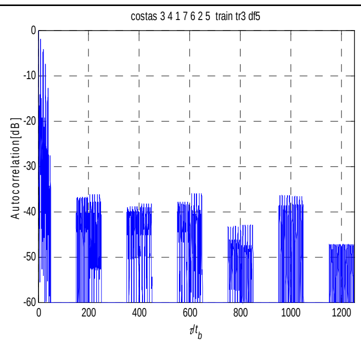
We applied Golomb ruler **0 3 4 12 18 23 25**, and Costas time spacing to the modified Costas sequence, with size **N=7,** frequencies **{3 4 1 7 6 2 5}**, frequency spacing $\Delta f = 5$ (In Costas sequence $\Delta f = 1/t_b$, but in modified Costad sequence $\Delta f = \alpha / t_b$ where α is integer number [8]. Each number of the Costas frequencies sequence multiplies by number α). The ACF over $\tau > t_b$ depends on the number of sub-pulses N, spacing between sub-pulses, and the order arrangements of frequencies. But for $\tau < t_b$ the order of frequencies does not affect ACF. Comparing between ACF of Costas sequence N=7, frequencies { 3 4 1 7 6 2 5}, and Golomb ruler 0 3 4 12 18 23 25 and $\Delta f = 5$ Fig. 10-b, with the ACF of Costas sequence N=7, frequencies { 3 4 1 7 6 2 5} Fig. 10-a, and with the ACF of Costas sequence N=7, frequencies { 3 4 1 7 6 2 5} and fix spacing between sub-pulses $T_r = 3t_b$, $\Delta f = 5$ Fig. 10-c, and with the ACF of Costas sequence N=7, frequencies { 3 4 1 7 6 2 5}, $\Delta f = 5$ and Costas variable spacing Fig. 10-d we notice that the Costas sequence with Golomo ruler spacing time has the minimum side-lobes at $\tau > t_b$.



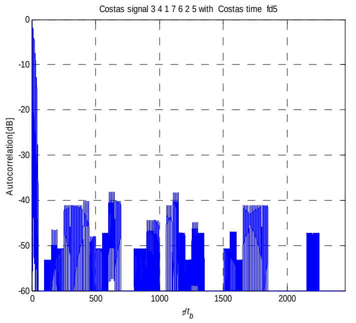
a. **ACF of Costas sequence N=7, {3 4 1 7 6 2 5}, and** $\Delta f = 5$.



b. ACF of Costas sequence N=7, {3 4 1 7 6 2 5} with Golomb ruler 0 3 4 12 18 23 25 and $\Delta f = 5$.



c. ACF of Costas sequence N=7, {3 4 1 7 6 2 5} with fix spacing, $T_r = 3t_b$, **and** $\Delta f = 5$.



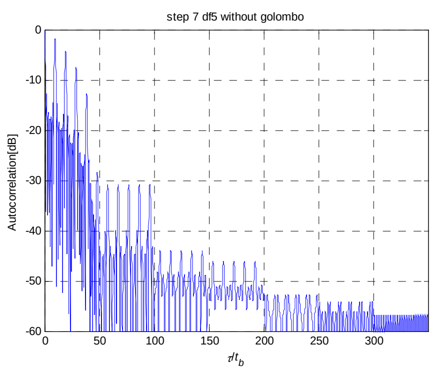
d. ACF of Costas sequence N=7, {3 4 1 7 6 2 5}, with Costas spacing, and $\Delta f = 5$.

Figure 10

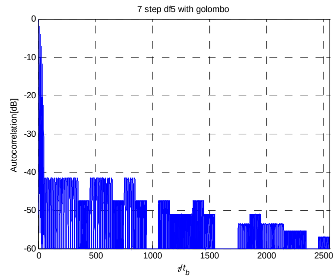
C. Frequency coded: Modified Stepped frequency signal.

We applied Golomb ruler **0 3 4 12 18 23 25**, and Costas time spacing **3 4 1 7 6 2 5** to the modified stepped frequency, with N=7, (where N number of sub-pulses) stepped frequencies and frequency spacing $\Delta f = 5$. The ACF over $\tau > t_b$ depends on the number of pulses N, spacing between pulses. The ACF at $\tau < t_b$ is the same as par. 5-b because the order of frequency does not affect the ACF over $\tau < t_b$.

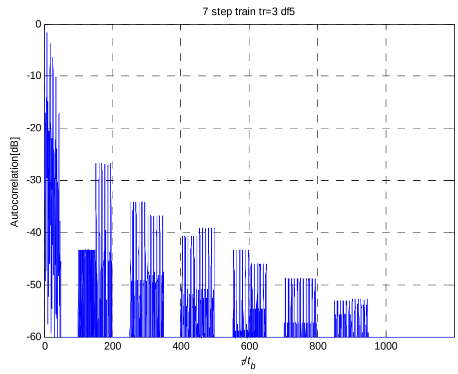
Comparing between the ACF of modified stepped frequency $\Delta f = 5$ and Golomb ruler 0 3 4 12 18 23 25, and $\Delta f = 5$ Fig. 11-b, with the ACF of modified stepped frequency, $\Delta f = 5$, without spacing Fig. 11-a, and with the ACF of modified stepped frequency and fix spacing $T_r = 3t_b$, $\Delta f = 5$ Fig. 11-c, and with the ACF of modified stepped frequency and Costas variable spacing and $\Delta f = 5$ Fig. 11-d we notice that the Costas sequence with Golomo ruler spacing has the minimum side-lobes at $\tau > t_b$.



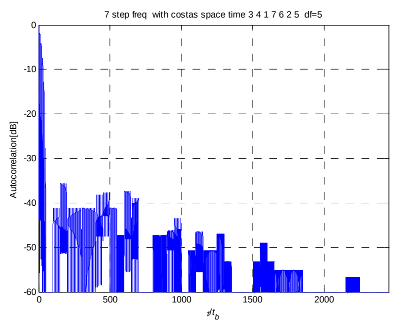
a- ACF of modified stepped frequency with $\Delta f = 5$.



b- ACF of modified stepped frequency $\Delta f = 5$ with Golomb ruler 0 3 4 12 18 23 25, and $\Delta f = 5$.



c- ACF of modified stepped frequency with fix spacing, $T_r = 3t_b$, and $\Delta f = 5$.



d- ACF modified stepped frequency with Costas spacing, and $\Delta f = 5$.

Figure 11

6- Results and Conclusion:

Adding variable spacing between the sub-pulses of **Costas signal** at any frequency coded allows decreasing side-lobes at $\tau > t_b$, and improving the Doppler resolution. The delay resolution of frequency coded is the same as in normal signal without adding variable spacing (Costas sequence $1/N^2$). Also there is no need to increase the size N of frequency coded or increase the frequency spacing.

Variable spacing, which achieves the previous properties, can be done by arranging the sub-pulses of the costas signal in time according to the following conditions:

1. The minimum space time between sub-pulses should be more than $2t_b$.
2. The space between sub-pulses should be different from one sub-pulse to another sub-pulse.
3. At range $\tau > t_b$ there are no more than one or two sub-pulses of received signal cross the reference signal.

These requirements are achieved by using Golomb rulers or binary Costas array upon frequency coded signals.

In addition we find that using Golomb rulers for variable spacing is slightly better than the Costas array.

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