Improving Ambiguity Function of Costas Signal

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***Abstract*— This paper presents the effects of modifying Costas signal in time and frequency to improve Ambiguity Function (AF). The effect of adding frequency spacing, linear frequency modulation, constant space time, and variable space time between sub-pulses of Costas signal are presented and compared. It is shown that a considerable reduction of side-lobe and better Doppler resolution is achieved by using variable spacing between sub-pulses of Costas signal.**

Keywords: Ambiguity Function; Costas signal; Pulse Compression.

INTRODUCTION

For good detection of radar signal a ratio of a large peak signal power to average noise power is needed, at the time of the target's return signal. As we know the matched filter is the best of all possible filters which it produces the maximum ratio.

This maximum ratio depends on the total transmitted energy and not on the presence of any frequency modulation on the transmitted signal. Thus for good detection many radars seek to transmit long-duration pulses to achieve high energy, since transmitters are typically operated near their peak power limitation.

On the other hand, for good range measurement accuracy, radar needs short pulses. These divergent of the needs of long pulses for detection and short pulses for range accuracy in measurements prevented early radars from simultaneously performing both functions. Fortunately in the late 1950s and early 1960s a new concept was developed whereby both needs could be met. The concept is called "pulse compression" [5]. It makes use of the fact that a long-duration pulse's bandwidth can be made larger by use of modulation in frequency or phase. Large bandwidth implies narrow effective duration. With modulation a waveform can be designed to have both long pulse duration and short pulse effective duration (large bandwidth). The waveform with short pulse effective duration is produced when the long-duration waveform with modulation is applied to its matched filter. Thus, by use of modulation in frequency or phase over long transmitted pulses and a matched filter, a system can simultaneously obtain good detection performance and accurate range measurements Fig.1.

The researchers develop many radar signals assisted by modern signal processing systems. Consequently, signals in different shapes have presented like Phase coded signals such as Barker Code, Frank Code, P1, P2, P3, Px Codes, as well as m-sequence code, Colombo code,… et, and frequency coding such as Linear Frequency modulation (LFM), non linear frequency modulation (NLFM), Stepped frequency modulation, and Costas signal [2,3,7,10].

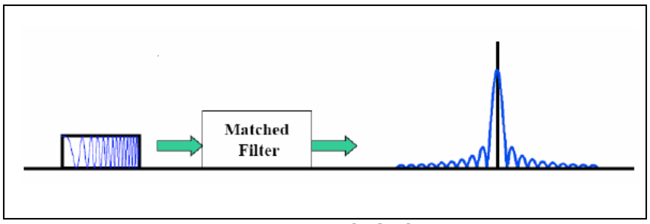

Figure 1. Matched Filter

Each of these signals has advantages and disadvantages. But the one of the most important one in the frequency coding is Costas signal.

Costas signal, with pulse-width T, consists of N sub-pulses. Each sub-pulse has different frequency as shown in Fig.2 [2].

Each frequency is chosen from a series of frequencies within the Bandwidth B. We have N frequencies, and pulse width of each sub-pulse is given by $t_b = T/N$. Costas has suggested algorithm to arrange the frequencies to enable us to control the side-lobes in such a way that these side-lobes will not exceed 1/N [6]. Then, the biggest side-lobe in ambiguity function (AF) is 1/N of its value in the main lobe Fig.3. Costas signal has a delay resolution of $1/N^2$ of pulse-width and the Doppler resolution is 1/T, and because of using matched filtering in the receiving system this signal has noise immunity. However, Costas signal is not an Ideal signal [1].

The objective of our research is studying the effects of the change of frequency spacing Δf where $\Delta f = 1/t_b$ upon Costas Signal, adding LFM within each sub-pulse, and inserting a separate time between sub-pulses of Costas signal.

This study has lead to improve ambiguity function by decreasing the side-lobes and increasing delay resolution and Doppler resolution without increasing the size of Costas array.

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This leads to the enhancement of performance specifications without the need to increase Costas sequence [4].

Therefore, we shall present in this paper a study of the effect of modifying Costas signal and uses the ambiguity function (AF) as a means of visualizing this signal as follows .

- Increasing the frequency spacing $\Delta f > 1/t_b$ of Costas sequence.
- Adding LFM to individual sub-pluses of Costas sequences $t_b * B$.
- Both of above mentioned items together.
- Adding Constant space time between sub-pulses of Costas sequence.
- Adding variable time spacing between sub-pulses of Costas sequence.

In our study we will use Costas sequence {3 6 2 7 5 4 1 8}, as an example, $\Delta f = 1/t_b$ (Similar results is obtained from other Costas sequence of length 8).

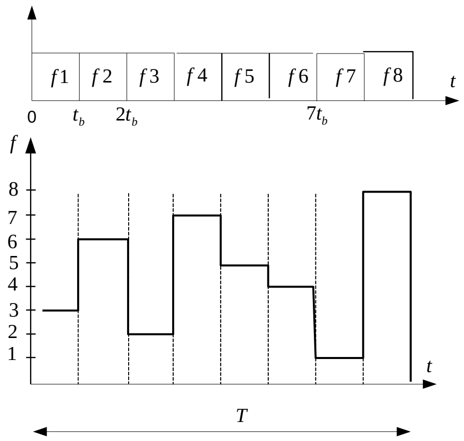


Figure 2. Costas signal

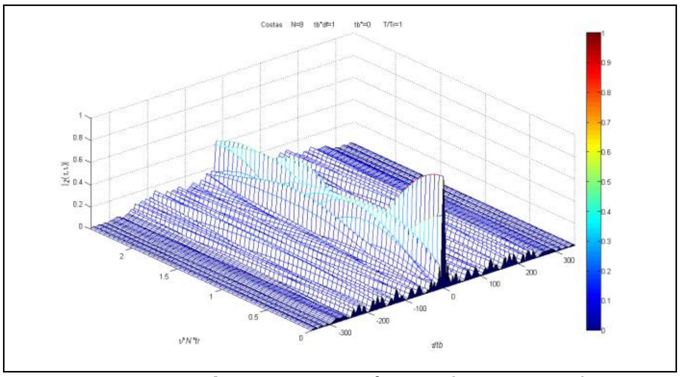


Figure 3. Ambiguity Function of Costas {3 6 2 7 5 4 1 8}

I. Study the effect of increasing the frequency spacing $\Delta f > 1/t_b$ of Costas signal.

The frequency spacing parameter Δf was increased without changing any other parameters and plotting the AF, measuring the maximum value of side-lobes at $\tau > t_b$, and measuring the width of the main lobe at zero Doppler cut of AF, for frequency spacing of $t_b * \Delta f$ equal 1 to 8.

TABLE 1. Results of the increasing the Frequency Spacing $\Delta f * t_b$ of Costas Signal

N	$t_b * \Delta f$	width of the main lobe	Max. value of side-lobes at $\tau > t_b$
8	1	$t_b/8$	-16dB
8	2	$t_b/15$	-22dB
8	3	$t_b/22$	-25dB
8	4	$t_b/29$	-30dB (fig3,4)
8	8	$t_b/64$	-34dB

From table 1 and the plots of AF (Fig 4 and 5), we notice the following:

- Great decrease of side-lobes at zero Doppler Cut of AF by increasing the frequency spacing within the range $\tau > t_b$. Notice the maximum value of side-lobes at $\tau > t_b$ for $t_b * \Delta f = 4$ is -30dB.
- The Time-Bandwidth product for our signal is given by
  $$T(f_{\max} - f_{\min}) = N(N-1)t_b\Delta f + N .$$
  Comparing to N^2 for Costas Signal.
  Therefore the delay resolution which is directly proportional to Time-Bandwidth product is increased by increasing the frequency spacing of Costas Signal.
- Increasing the frequency spacing reveal grating lobes and side-lobes at $\tau < t_b$. The number of grating lobes is increased by increasing the frequency spacing of Costas signal.

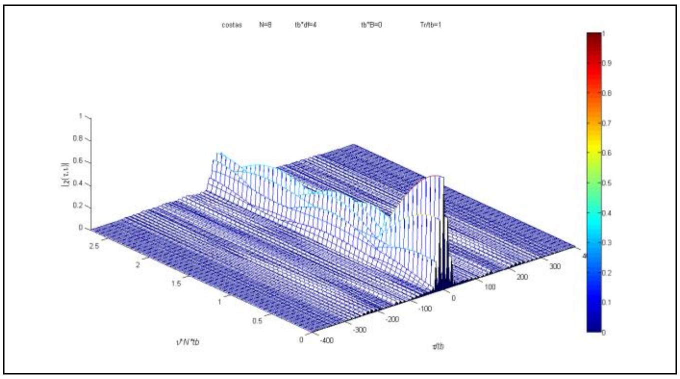


Figure 4. AF of Costas N=8, $t_b * \Delta f = 4$, $t_b * B = 0$, $T_r / t_b = 1$

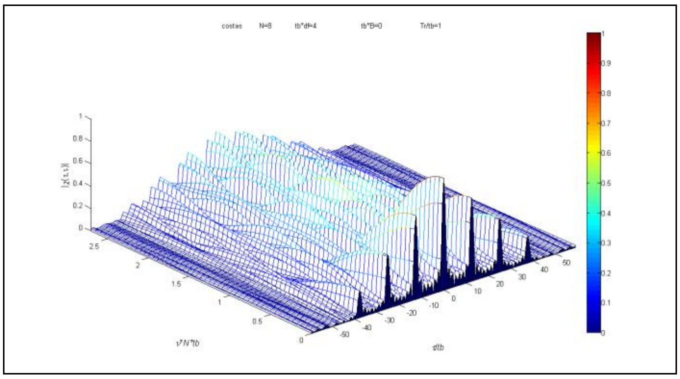


Figure 5. AF of Costas N=8, $t_b * \Delta f = 4$, $t_b * B = 0$, $T_r / t_b = 1$ at $\tau < t_b$

II. Studying the effect of adding LFM to individual sub-pluses of Costas sequences $t_b * B$.

In this study we add LFM to each sub-pulses of Costas signal. The bandwidth of each sub-pulse is increased from $t_b * B = 1$ to $t_b * B = 8$.

The following table shows the measurement of the first recurrent lobe at $\tau < t_b$, and the maximum value of the side-lobes at $\tau > t_b$.

TABLE 2. Results of the change of band width of sub-pluses $t_b * B$ of Costas Signal

N	$t_b * B$	$t_b * \Delta f$	first recurrent lobe $\tau < t_b$ (dB)	Max. value of the side-lobes $\tau > t_b$ (dB)
8	0	1	-16.85	-18.85
8	1	1	-17	-18.18
8	2	1	-18	-17.2
8	3	1	-21	-16
8	4	1	-25	-16.22
8	5	1	-34	-16.17
8	8	1	-28.56	-16.17

We notice from table 2 by adding LFM to the sub-pulses of the Costas signal that the main effect is reduction of values of the side-lobes of AF at Zero Doppler Cut ($\nu = 0$) at $\tau < t_b$. It is spread the AF at zero Doppler cut near the main-lobe at $\tau < t_b$ along the range $\tau > t_b$.

III. Studying the effect of increasing the frequency spacing and adding LFM to individual sub-pluses of Costas Signal.

In this study we have placed fixed frequency spacing $t_b * \Delta f = 4$ and adding LFM to the sub-pulses. The LFM is varied from a value $t_b * B = 4$ to the value $t_b * B = 8$, and we find the following (See Fig. 6 and table 3):

Reduction the maximum value of the first recurrent lobe of AF at zero Doppler cut ($\nu = 0$). (See fig 6 and compared with Fig. 5).

For a given value of $t_b * \Delta f$ there is an optimum value $t_b * B$, that achieves a good AF therefore there is relation between the frequency spacing and the LFM which has lead to the reduction of the grating lobe and the side-lobe at $\tau < t_b$ and reduction the side-lobe at $\tau > t_b$ to acceptable values.

TABLE 3. Results of adding fixed frequency spacing and adding LFM to sub-pulses

N	$t_b * B$	$t_b * \Delta f$	Max. value of the side-lobes $\tau > t_b$ (dB)
8	1	4	-29.2
8	2	4	-29.78
8	3	4	-29.59
8	4	4	-27
8	5	4	-24
8	6	4	-21.14
8	7	4	-18.56

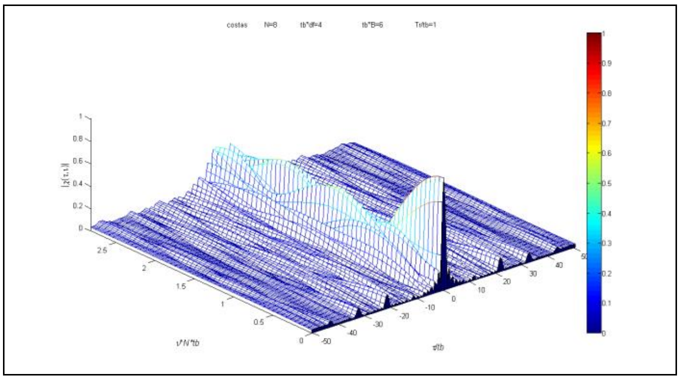


Figure 6. AF of Costas N=8, $t_b * \Delta f = 4$, $t_b * B = 6$, $T_r / t_b = 1$ at $\tau < t_b$

IV. Studying the effect of adding constant space time between sub-pulses of Costas signal.

In previous sections, we added frequency spacing and LFM to the sub-pulses. This improves the delay resolution and depressed the side-lobes. In spite of that, the recurrent side-lobe level upon $\tau > t_b$ is not neglected. The recurrent side-lobe level is depend on the number of cross correlation frequencies at any time. If we reduce the numbers of frequency cross correlation, we can decrease the recurrent side-lobe at $\tau > t_b$ of the Costas signal. To achieve that, we will add fixed space time between sub-pulses.

In this study we have added constant space time between sub-pulses to the Costas signals with duration $T_r > 2t_b$, and the fixed frequency spacing was varied from a value $T * \Delta f = 1$ to value $T * \Delta f = 5$. We find (table 4) that the maximum recurrent side-lobe of Costas signal at $\tau > t_b$ is reduced at least 2dB when added constant space time, compared with the side-lobe of AF at zero Doppler cut (Autocorrelation Function (ACF)) of the same signal without constant space time between sub-pulses for deferent frequency spacing.

TABLE 4. Results of adding constant space time between sub-pulses of Costas signal

Max. value of the side-lobes (dB) $\tau > t_b$			
N	$t_b * \Delta f$	Frequency spacing	Frequency spacing & Time spacing
8	1	-16.3	-19.62
8	2	-22.29	-25.51
8	3	-25.82	-29.07
8	4	-28.34	-31.61
8	5	-30.29	-33.47

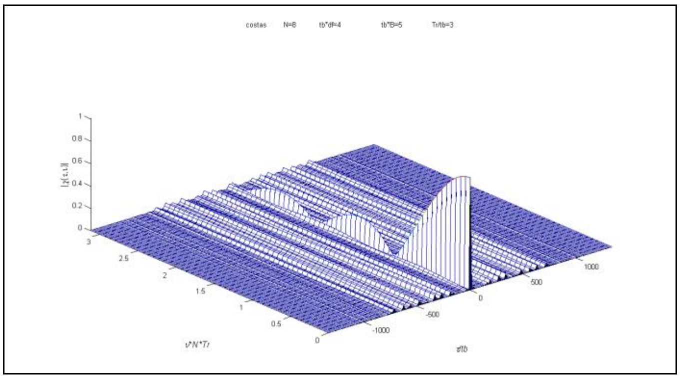
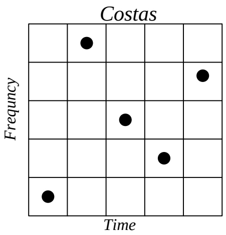


Figure 7. AF of Costas N=8, $t_b * \Delta f = 4$, $t_b * B = 5$, $T_r / t_b = 3$

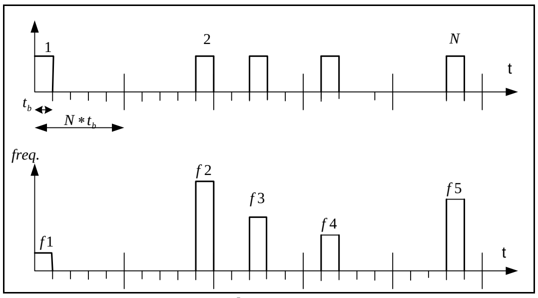
V. Studying the effect of adding variable time spacing between sub-pulses of Costas signal.

In this section we will try to reduce cross correlation to one or two number of frequency cross correlation at any time of ACF of the signal, by using Costas Array (Fig.8-a). Costas Array has only one point at each column and row and the two-dimensional discrete autocorrelation function of array exhibit only "1" and "0" values, except at the origin where autocorrelation value is N.

We have placed variable spacing between sub-pulses by spreading Costas Array $N*N$ in time. We put the first column, second column,…et, each column is apart of the signal, each part have pulse (Pulse width t_b, frequency f_n.) placed according to the Costas signal, (the signal time is $N*N*t_b$) example $t_b = 2$, $f_n = 5$, that is means, at part two of the signal there is pulse at slice 5 with frequency $f_n = 5$ as shown in Fig. 8-b .



a.



b.

Figure 8. Costas signal with Costas time spreading.

Comparing the AF of Costas signal, with the AF of Costas signal with constant space time and with AF of Costas signal with variable space time, we find the following:

- Decrease the side-lobes of AF at zero Doppler cut within the range $\tau > t_b$ at Costas signal with variable space time more than 5dB than the side-lobe of ACF of the same signal with fixed space time between sub-pulses or Costas signal. (table5).
- The delay resolution of Costas signal with variable space time is the same as in normal Costas signal $1/N^2$
- The time signal of Costas signal with variable space time is larger than the normal Costas signal. Then the Doppler resolution is increased by factor N

TABLE 5. Results of adding variable time spacing between sub-pulse of Costas array

Max. value of the side-lobes $\tau > t_b$			
Costas	Normal Costas (dB)	Costas Train (dB)	Costas with variable time spacing (dB)
2-1-6-4-7-3-5	-16.49	-18.97	-24.82
2-1-5-7-3-6-4	-.16.96	-19	-24.75
3-6-1-7-5-2-4	-16.98	-18.92	-25
2-4-7-3-1-6-5	-18.64	-21.99	-24.74
3-4-1-7-6-2-5	-19.97	-21.99	-24.75

VI. Discussion and Conclusion

We showed two methods of improving of the ambiguity function (AF) of Costas signal first method: By increasing the frequency spacing and Adding LFM for each sub-pulse, yield to a reduction of side-lobes upon $\tau > t_b$, and a reduction the grating lobes at $\tau < t_b$, in addition to that the bandwidth become wider and the delay resolution is increased. But this method makes radar design more complicated.

Second method: By adding variable spacing between sub-pulses of Costas signal, yield better side-lobes (at $\tau > t_b$) and Doppler resolution when compared with the first method, and there isn't grating lobes to be solved. The radar system is easier to design, but this is achieved at the expense of increasing transmitting time.

If we apply both methods together to Costas signal we get the advantages of both methods.

REFERENCES

[1] Rihaczek, A. W., “Principles of High Resolution Radar”, McGraw-Hill, New York, 1969.

[2] Costas, J. P., “A study of a class of detection waveforms having nearly ideal range-Doppler ambiguity properties”, *Proceedings of the IEEE*, vol. 72, no. 8, August 1984, pp. 996-1009.

[3] Golomb, S. W., and H. Taylor, “Constructions and properties of Costas arrays”, *Proceedings of the IEEE*, vol. 72, no. 9, September 1984, pp. 1143- 1163.

[4] Levanon, N., Mozeson, E(2003) “Nullifying ACF grating lobe stepped-frequency train of LFM pulses”. *IEEE Transaction on Aerospece and Electronics Systems*, 39, 2 (Apr.2003), 694-703

[5] Cook,C.,E. and M. Cook, C. E., and W. M. Seibert, “The early history of pulse compression radar”, *IEEE Transactions on Aerospace and Electronic Systems*, vol. AES-24, no. 6, November 1988, pp. 825-837.

[6] Scott Rickard, Claude Shannon Institute, University College Dublin, “Open Problems in Costas signal”, December 19,2006.

[7] T.D Bhatt, E.G. Rajan, P.V.D. Somasekhar Roa “Design of frequency-coded waveforms for target detection” IET Radar Sonar Navig., 2008, Vol. 2, NO. 5, pp. 388-394

[8] Levanon, N., Mozeson “Radar Signals” 2004

[9] Levanon, N., “Radar Principles”, Wiley, New York, 1988.

[10] James K. Bread, Jon C. Russo, Keith G. Erickson, Michael C. Monteleone, Michael T. Wright; “Costas Array Generation and Search Methodology” *IEEE Transaction on Aerospece and Electronics Systems*, 43, NO. 2 (Apr.2007), 522-538